3.4.9. Energy Balance

In this paragraph we will compute the yearly thermal energy flows into and out of the cave and try to find the net balance. Let us again stress that we assume that the cave has only one single entrance in the main storey: this means that in summer all the air will enter the cave through the main entrance, and in winter all outflow will be through this same opening. The air entering the cave brings a certain enthalpy and kinetic energy to the underground system. As the air velocities are low, we can readily neglect the kinetic energy in comparison to the enthalpy.

We first will compute an expression of the enthalpy of 1 m^3 moist air entering the cave, following a reasoning given by Badino [Badino, 1995].

Let us start by a mass of moist air consisting of 1 kg dry air and d kg water vapour (so the total mass is m = 1+d). The enthalpy of this mixture consists of 3 parts:

- H1 = enthalpy of 1 kg dry air at T °C = c_p*1*T = 1007* T [J]

    1007 [$J*kg^{-1}*K^{-1}$] = specific heat of dry air

- H2 = evaporation energy corresponding to d kg water vapour,
which will be restored when condensation occurs

    = $2501*10^3*d$ [J]

    2501000 [$J*kg^{-1}$] = latent heat of water vapour

- H3 = enthalpy of d kg water vapour at T °C = 1930*d*T [J]

    1930 [$J*kg^{-1}*K^{-1}$] = specific heat of water vapour

The total enthalpy in Joule of m = 1+d kg moist air is :

$$H_m = H_1 + H_2 + H_3$$
$$H_m = 1007 * T + d * \left[2.501*10^6 + 1930 * T\right] \qquad \text{[eq. 3.4.13]}$$

It can be shown [Recknagel, 1992; Badino, 1995] that

$$d = 6.22*10^{-4} * p_V \qquad \text{[eq. 3.4.14]}$$

where p_V is the water vapour pressure in [hPa]; p_V is equal to (H%/100)*p_{sat} where H% is the relative humidity in percent and p_{sat} the saturated water vapour pressure in [hPa]. For the temperature range valid in the caves, we may use the following good approximation to p_{sat} [Choppy, 1992], as seen in chapter 2:

$$p_{sat} \cong \left(\frac{T}{6} + 1\right)^2 + 5 \qquad \text{[eq. 3.4.15]}$$

which finally allows to express d as a function of T and H%:

$$d \cong 1.728 * 10^{-7} * H\% * \left[(T+6)^2 + 180\right] \qquad \text{[eq. 3.4.16]}$$

H_m represents the enthalpy of m = (1+d) **kg** of moist air; the enthalpy H_v of 1 **m³** of moist air is equal to H_m divided by the density of moist air.

Using the expression of this density found in chapter 2, we can express the enthalpy H_v as a function of the **measurable** parameters air temperature T, relative humidity H% and air pressure p:

$$H_v = \frac{1007 * T + 1.728 * 10^{-7} * H\% * \left[(T+6)^2 + 180\right] * (2.501 * 10^6 + 1930 * T)}{\frac{0.34855}{273.16 + T} * \left[p - \frac{0.406}{3600} * H\% * \left((T+6)^2 + 180\right)\right]}$$

[eq. 3.4.17]

where

T air temperature [°C]

H% relative humidity [percent]

p air pressure in [hPa]

H_v enthalpy of 1 m^3 of moist air [J]

The dominant factor influencing H_v is temperature; actually the curves of H_v and T vary almost identically, as shown by the next figure 3.4.14. which plots H_v (upper graph) and T (lower graph) at station 2 in Dec.93.

If the air enters the cave with a velocity v [$m*s^{-1}$] through a gallery-section of S m^2, we can compute the total energy entering the cave during one hour by:

*Energy = Hv**v**0.74*S*3600/3600000 = Hv**v**0.74*S/1000 [kWh]*

[eq. 3.4.18]

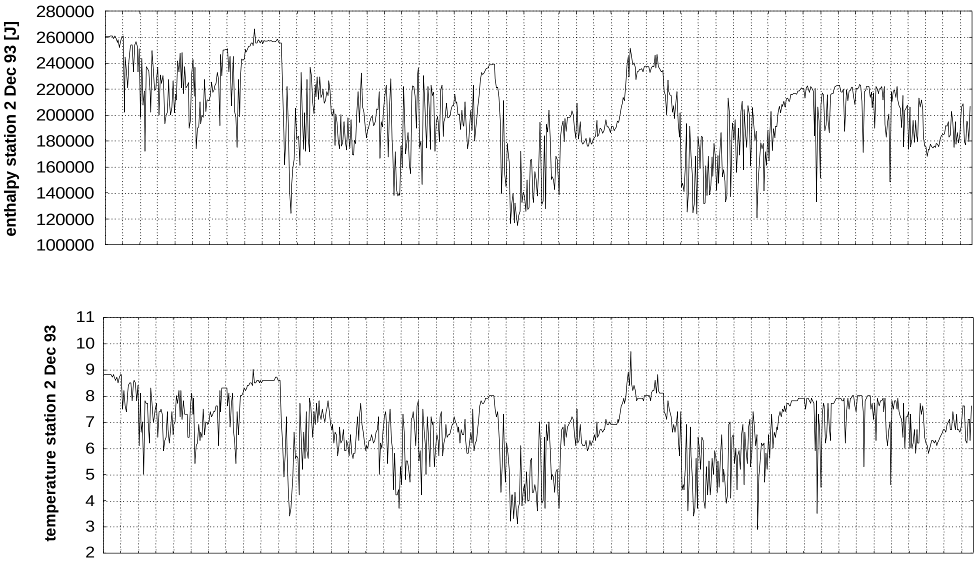

Fig. 3.4.14. Variations of enthalpy and external temperature in Dec.93(subset;ticks mark hours)

The multiplier 0.74 is used to compute an approximate mean velocity over the whole section from the centre velocity v (factor valid for very rough pipes) [Parker, 1988].

The balance over a whole month can be computed as follows: Independently of the flow direction, air enters the cave during all the time, either through the single entrance of the main gallery or from the lower situated galleries. The corresponding energy E_{in} is computed by using the external temperature, relative humidity and air pressure, and the velocity v at station 2 (multiplied by 0.74): $E_{in} = H_v(T_{ext},H_{ext}\%,p,v)$. All the air which enters the cave must leave it again; we will assume that during its journey in the cave this air has taken the mean cave temperature of 9.4°C and will become saturated at 100% relative humidity, its pressure being the same as the outside atmospheric pressure; let E_{out} denote the energy leaving the cave: $E_{out} = H_v(9.4°C,100\%,v,p)$.

The net energy balance is:

$$DE = E_{in} - E_{out} = H_v(T_{ext},H_{ext}\%,v,p_{ext}) - H_v(9.4,100,v,p)$$

[eq. 3.4.19]

Table 3.4.9 shows the results for a whole year, starting in December 1993 and ending November 1994.

Table 3.4.9

month	*Ein [Kwh]*	*Eout [Kwh]*	*DE [KWh]*
Dec93	18457	30256	-11799
Jan94	13596	25684	-12088
Feb94	5945	16851	-10906
Mar94	18986	25094	-6108
Apr94	11011	13123	-2112
May94	24901	22735	+2166
Jun94	35868	27877	+7991
Jul94	49294	29079	+20215
Aug94	46653	29079	+17574
Sep94	27668	21615	+6053
Oct94	16853	16467	+386
Nov94	16323	15852	+471
Totals	285557	272980	+12577

The net balance over 12 month is:

- Dec91--->Nov92: -22050 (estimated) [KWh]
- Dec92--->Nov93: -24722 [KWh]

The negative balance means that the cave actually lost energy to the outside during these two years, whereas it gained 12577 [KWh] during the period from Dec. 1993 to Nov. 1994.

The autumn months are those of minimal energy transfer (and corresponding minimal perturbation of the underground system), and the two summer months of July and August do usually inject into the cave more or less the same amount of thermal energy than it looses during the following two cold winter months of December and January:

- 1992: inflow Jul.92 + Aug.92 = 61332 [KWh]
  outflow Dec.92 + Jan.93 = 57935 [KWh]

- 1993: inflow Jul.93 + Aug.93 = 56265 [KWh]
  outflow Dec.93 + Jan.94= 55940 [KWh]

The correlation between the monthly energy balance ΔE and the mean monthly outside temperature T_m is very high: r=0.97!

The following two figures (fig. 3.4.15 & 3.4.16) show the seasonal variations of these two parameters and the linear fit of ΔE to T_m . Actually the linear fit gives a good model to estimate the monthly energy balance from the mean external temperature, without resorting to the complicated calculations involving H%, v and p. According to the model, the reversal temperature that gives a zero energy balance is 10.7°C, about 1.3°C above the measured mean cave temperature of 9.4°C.

If we force the linear fit through (T_{ext} = 9.4°C; dE = 0) we get the following equation:

*dE @ 1442*T - 1644* with dE in [KWh] and T in [°C] *[eq. 3.4.20]*

Let us insist again on the various simplifying assumptions made in this chapter; we have for instance completely neglected that oscillating air movements may not extend throughout the whole cave. The computed values of E_{in} and E_{out} should be seen as rough and probably too high estimates.

The contribution of the geothermal heat flux to the energy balance of the cave is negligible; if we assume a mean geothermal flux of 0.0418 W*m^{-2} [Badino, 1995], a total gallery length of 3500 m with a mean width of 0.5m, we will get for the whole cave a geothermal flux of 73 W, which corresponds to a monthly energy of 53 KWh, about 2 to 3 order of magnitudes smaller than the energy E_{in} or E_{out} computed above.

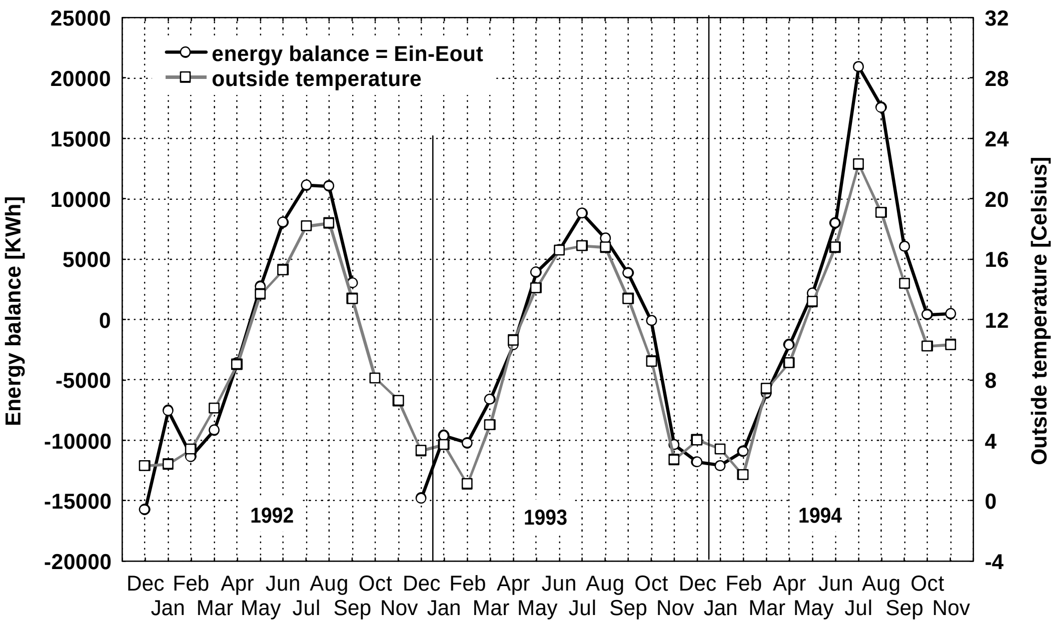

Fig. 3.4.15. Seasonal pattern of energy balance and outside temperature